

# E39B Homework 3

*due 09 Mar*

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The Monte Carlo method can be used to integrate a function  $y = f(x)$  with the following technique:

1. choose a uniformly distributed random number  $x$  along the interval of interest  $x_{\min} \leq x \leq x_{\max}$
2. choose a second uniformly distributed random number  $y$  along the interval of interest  $y_{\min} \leq y \leq y_{\max}$
3. compare:  $y \leq f(x)$  each time it is true, add 1 to  $s$
4. go back through loop starting at (1) until  $n$  iterations are done
5.  $area = (x_{\max} - x_{\min})(y_{\max} - y_{\min})s/n$

Note that the function should be bound by  $x_{\min}, x_{\max}, y_{\min}, y_{\max}$ , but need not be continuous or differentiable.

The example file on the course web page, MCexample.m, integrates the area under a diamond formed by folding the line  $y = 1 - x$  over each axis. The area is computed for 1/4 the system (for  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ ), and the final result is multiplied by 4. Hence, the exact answer is 2. Note that the above technique has been modified to take advantage of MatLab's array capabilities, which greatly improves the performance over a straight *for*-loop implementation of the above algorithm. You can think of this Monte Carlo technique as throwing darts at a unit square, and finding out how often the dart lands inside a line defined by  $y = f(x)$ .

Modify the MCexample.m file to calculate  $\pi$ , noting that it is given by the area of a circle of radius 1, i.e.  $y = \sqrt{1 - x^2}$  between  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ , multiplied by 4 (because the preceding equation is only 1/4 of a circle in area). Calculate  $\pi$  for  $10^n$  points, where  $n = 1, 2, 3, 4, 5, 6$ . Show the results for each calculation, as well as a graph of the errors compared to MaLab's built-in  $\pi$ , given by the built-in variable *pi*.